

Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

140. Proposed by PROF, R. D. CARMICHAEL, Anniston, Ala.

Determine (any way) whether the Diophantine equation $\left(\frac{2x-1}{3}\right) = x^2 + y^2$ has any positive integer solutions.

Solution by JACOB WESTLUND, Ph. D., Purdue University.

In order that $\frac{2x-1}{3}$ shall be an integer we must have x=2+3a, where a is a positive integer. Hence $(1+2a)^3=(2+3a)^2+y^2$ or after a few reductions $y^2=8a^3-6a+3(a^2-1)$.

If a is odd, this equation is impossible, since in that case y must be even and hence all the terms except 6a divisible by 4.

If a is even, we put the equation in the form $y^2 = 8a^3 + 3a(a-2) - 3$. This shows that y must be odd and y + 3 divisible by 8. Hence, setting y=2b+1, $4b^2+4b+4$ should be divisible by 8 or b(b+1)+1 divisible by 2, which is impossible. Hence the given equation has no positive integer solutions.

Also solved by A. H. Holmes.

No solution of 141 has yet been received.

AVERAGE AND PROBABILITY.

178. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

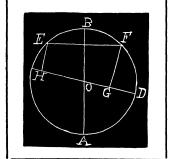
Two random planes cut a given sphere. What is the chance that they intersect within the sphere?

I. Solution by HENRY HEATON, Belfield, N. D.

Let AB and CD be axes of the sphere perpendicular to the two planes and let EF be a trace of one of the planes.

Put x=OI, the distance of the plane through EF from the center of the sphere. Put $\theta = \angle BOC$. Then HG, the projection of EF upon $CD=2_1 \angle (a^2-x)\sin \theta$.

It seems to be generally understood that the number of directions of the plane perpendicular to CD depends upon the number of different directions possible to CD, and that this depends upon the number of points in the surface of the sphere. Hence the number of planes



of the direction θ is proportional to $\sin \theta$. The angle θ being supposed fixed the chance of intersection within the sphere is $\frac{HG}{CD} = \frac{\sqrt{(\alpha^2 - x^2)\sin \theta}}{a}$.

Hence the required probability is $p = \int_0^{\frac{1}{2}\pi} \int_0^a V(a^2 - x^2) \sin^2\theta \ d^{\eta} \ dx \ / \int_0^{\frac{1}{2}\pi} \int_0^a a \sin\theta \ d^{\theta} \ dx$